Exercise 6

Using the fact that $|z_1 - z_2|$ is the distance between two points z_1 and z_2, give a geometric argument that

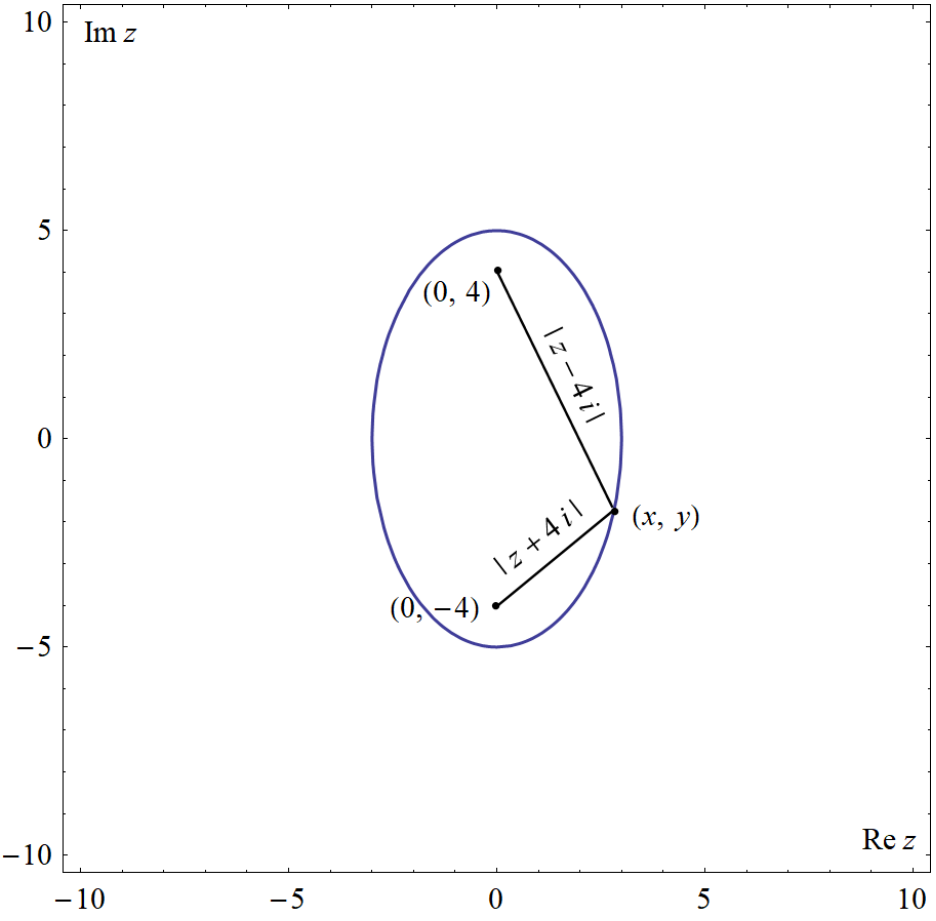
(*a*) $|z - 4i| + |z + 4i| = 10$ represents an ellipse whose foci are $(0, \pm 4)$;

(*b*) $|z - 1| = |z + i|$ represents the line through the origin whose slope is -1.

Solution

Part (a)

Because the sum of distances from two fixed points, $z = 4i$ and $z = -4i$, is a constant, the graph is an ellipse.



Part (b)

Because the distances from two fixed points, $z = 1$ and $z = -i$, are equal, the graph is a line.